Name:_____   Date:_____

Math 12 Enriched: Section 3.2 Factoring Polynomial Functions

1. Find the Quotient, remainder and write the division statement:

a) $y = x^2 - 5x + 2 \div (x-3)$	b) $y = x^3 + x^2 - 10x + 8 \div (x-2)$	c) $y = x^3 + x^2 - 2x + 3 \div (x+1)$
d) $y = 3x^3 - 5x^2 + 2x + 8 \div (2x-1)$	e) $y = 2x^3 + x^2 + 4x - 7 \div (x-4)$	f) $y = x^4 - 3x^3 + 2x^2 - 5x - 1 \div (2x+3)$

2. Determine each value of k .

a) When $x^3 + kx^2 + 2x - 3$ is divided by x+2, the remainder is 1.	b) When $x^4 - kx^3 - 2x^2 + x + 4$ is divided by x-3, the remainder is 16.
c) When $2x^3 - 3x^2 + kx - 1$ is divided by x-1, the remainder is 1.	d) When $2x^4 + kx^2 - 3x + 5$ is divided by x-2, the remainder is 3.
e) When $x^3 + kx^2 - 2x - 7$ is divided by x+1, the remainder is 5.	f) When $kx^3 + 2x^2 - x + 3$ is divided by x+1, the remainder is 4.

3. How do you know if a binomial is a factor of a polynomial? Explain it using your own words:

4. Given $f(x) = 2x^3 - 5x^2 - x + 6$, which of the following binomials is a factor $f(x)$?

i) $(x+2)$ ii) $(x-1)$ iii) $(2x+3)$ iv) $(x+1)$ v) $(2x-3)$ vi) $(x+3)$ vii) $(x-2)$ vii $(2x-1)$

5. How do you determine which factors to divide your function by when converting it to factor form? Explain using your own words. Provide an example:

6. Use the factor theorem to convert each function to factored form:

$a) f(x) = 2x^3 - 3x^2 - 8x + 12$	b) $f(x) = 2x^4 - 15x^3 + 36x^2 - 35x + 12$
c) $f(x) = 20x^3 + 17x^2 - 40x + 12$	d) $f(x) = x^3 + 9x^2 + 26x + 24$
e) $f(x) = 2x^3 + x^2 - 25x + 12$	f) $f(x) = 2x^4 - 7x^3 + 9x^2 - 5x + 1$

7. When $kx^3 + mx^2 + x - 2$ is divided by $x-1$, the remainder is 6. When this polynomial is divided by $x+2$, the remainder is 12. Solve for "k" and "m".

8. Factor completely: $8r^2 + 6rs - 12rs - 9s^2$

9. If r_1, r_2, r_3, r_4 are the roots of $x^4 - 9x^2 + 2 = 0$, what is the value of $(1+r_1)(1+r_2)(1+r_3)(1+r_4)$?

10. If $f(2x) = x^2 + 4x + 1$, what are al values of "t" for which $f\left(\dfrac{t}{2}\right) = \dfrac{-11}{4}$, where "f" represents a function?

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11. There are two real values of "r" for which $x^4 - x^3 - 18x^2 + 52x + k$ has a factor of the form $x - r$. One of these values is $r = 2$. What is the other value of "r"?

12. For what rational number "c" do the equations $x^3 + cx^2 + 3 = 0$ and $x^2 + cx + 1 = 0$ have a common solution?

13. Determine the exact value of the only real root of the equation: $x^3 + 6x^2 + 12x + 24 = 0$. (No calculators!!)

14. Determine all the solutions to the systems of equations: $x^2 + y^2 + x + y = 12$ and $xy + x + y = 3$

15. One real root of the equation $x^8 + x^6 + x^4 + x^2 = 340$ is $x = 2$. What is the only other real root of this equation?

16. Let "r" be a root of $x^4 - x^3 + x^2 - x + 1 = 0$. What is the value of $r^{40} - r^{30} + r^{20} - r^{10} + 1$?

17. Solve the following inequalities:
    i) $x^4 - 10x^3 + 35x^2 - 50x + 24 > 0$ ii) $x^4 + 6x^3 - 13x^2 - 66x + 72 \le 0$

18. Challenge: Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \bullet R(x)$?

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Find the remainder when $x^{81} + x^{49} + x^{25} + x^9 + x$ is divided by $x^3 - x$.

The polynomial $p(x)$ satisfies $p(-x) = -p(x)$. When $p(x)$ is divided by $x - 3$ the remainder is 6. Find the remainder when $p(x)$ is divided by $x^2 - 9$.